INSTITUTO DE FÍSICA
Universidade Federal Fluminense
Universidade Federal Fluminense

Eletromagnetismo

Newton Mansur

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t} \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{J} = \sigma \vec{E}$$

$$\vec{E}.\vec{\nabla} \times \vec{H} = \vec{E}.\sigma\vec{E} + \epsilon\vec{E}.\frac{\partial \vec{E}}{\partial t} \qquad \vec{E}.\vec{\nabla} \times \vec{H} = \vec{\nabla}.(\vec{H} \times \vec{E}) + \vec{H}.\vec{\nabla} \times \vec{E}$$

$$\vec{E}.\vec{\nabla} \times \vec{H} = \vec{\nabla}.(\vec{H} \times \vec{E}) + \vec{H}.\left(-\mu\frac{\partial \vec{H}}{\partial t}\right)$$

$$\vec{\nabla}.(\vec{H} \times \vec{E}) - \mu\vec{H}.\left(\frac{\partial \vec{H}}{\partial t}\right) = \vec{E}.\sigma\vec{E} + \epsilon\vec{E}.\frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla}.(\vec{H} \times \vec{E}) = \sigma E^2 + \frac{1}{2}\epsilon\frac{\partial E^2}{\partial t} + \frac{1}{2}\mu\frac{\partial H^2}{\partial t}$$

$$\vec{\nabla}.(\vec{H}\times\vec{E}) = \sigma E^2 + \frac{1}{2}\epsilon\frac{\partial E^2}{\partial t} + \frac{1}{2}\mu\frac{\partial H^2}{\partial t}$$

$$\int \vec{\nabla}.(\vec{E}\times\vec{H})\,dv = -\int \sigma E^2\,dv - \int \frac{1}{2}\epsilon\frac{\partial E^2}{\partial t}dv - \int \frac{1}{2}\mu\frac{\partial H^2}{\partial t}dv$$

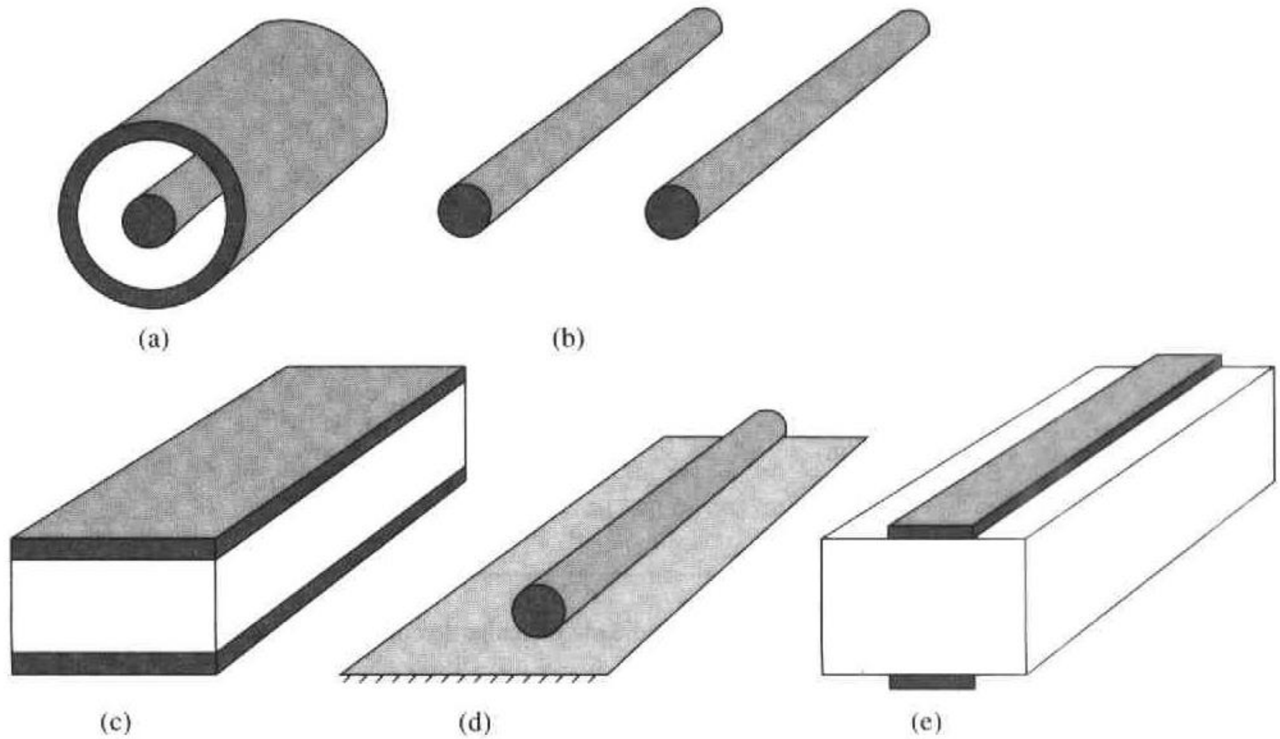
$$\oint(\vec{E}\times\vec{H}).d\vec{S} = -\int \sigma E^2\,dv - \frac{\partial}{\partial t}\int\left(\frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H^2\right)dv$$

$$\vec{S} = \vec{E}\times\vec{H} \quad \rightarrow \quad \textit{Vetor de Pointing}$$

Potência por unidade de área

Irradiância

Linhas de transmissão

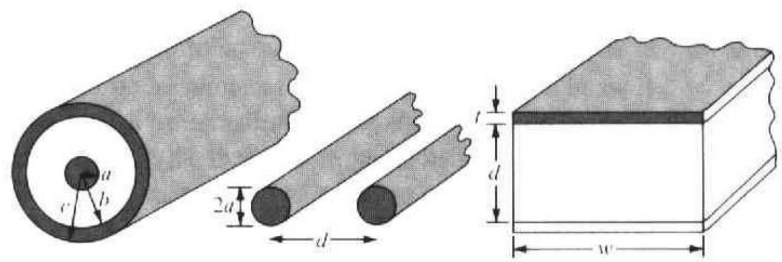


Linhas de transmissão

Parâmetros	**Linha Coaxial**	**Linha Bifilar**	**Linha Planar**
R (Ω/m)	$\frac{1}{2\pi\delta\sigma_c}\left[\frac{1}{a}+\frac{1}{b}\right]$ $(\delta \ll a, c - b)$	$\frac{1}{\pi a\delta\sigma_c}$ $(\delta \ll a)$	$\frac{2}{w\delta\sigma_c}$ $(\delta \ll t)$
L (H/m)	$\frac{\mu}{2\pi}\ln\frac{b}{a}$	$\frac{\mu}{\pi}\cosh^{-1}\frac{d}{2a}$	$\frac{\mu d}{w}$
G (S/m)	$\frac{2\pi\sigma}{\ln\frac{b}{a}}$	$\frac{\pi\sigma}{\cosh^{-1}\frac{d}{2a}}$	$\frac{\sigma w}{d}$
C (F/m)	$\frac{2\pi\varepsilon}{\ln\frac{b}{a}}$	$\frac{\pi\varepsilon}{\cosh^{-1}\frac{d}{2a}}$	$\frac{\varepsilon w}{d}$ $(w \gg d)$

$$LC = \mu\epsilon$$

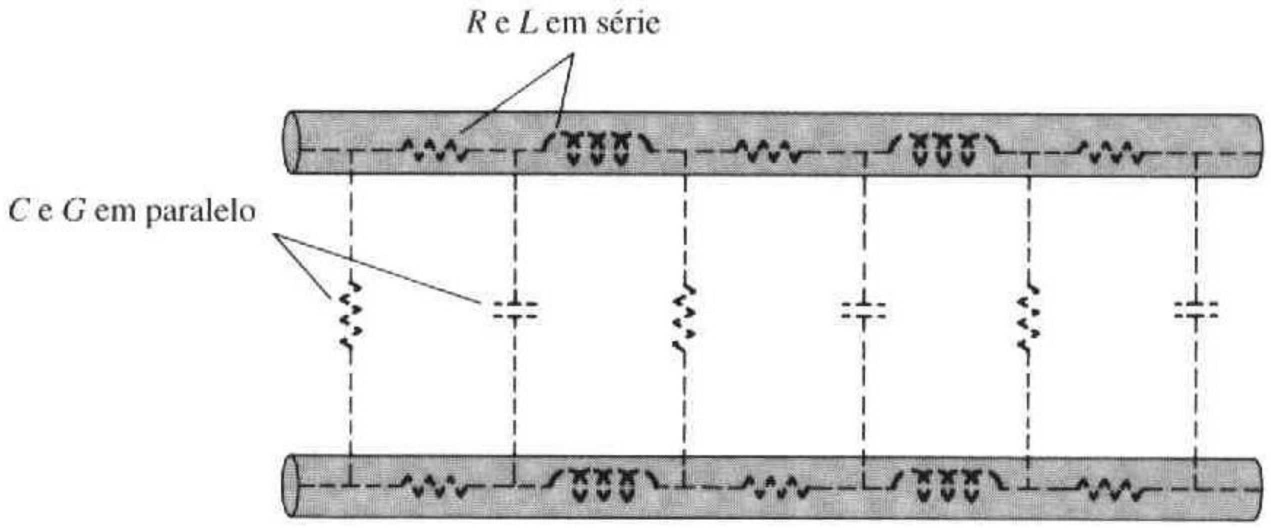
$$\frac{G}{C} = \frac{\sigma}{\epsilon}$$



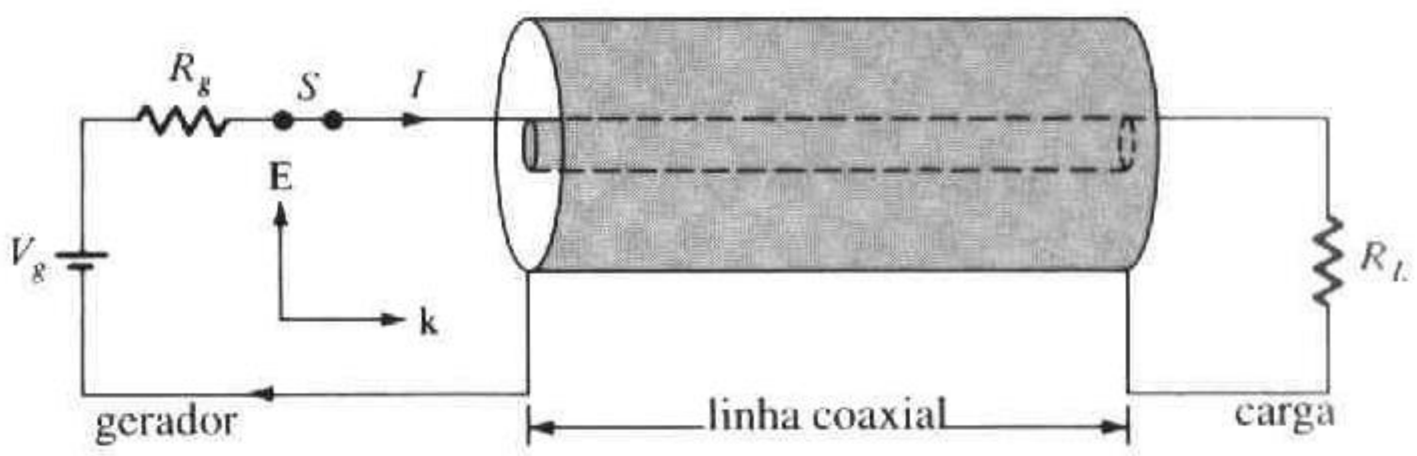
$$cosh^{-1}\frac{d}{2a} \sim ln\frac{d}{a} \quad se \quad \left(\frac{d}{2a}\right)^2 \gg 1$$

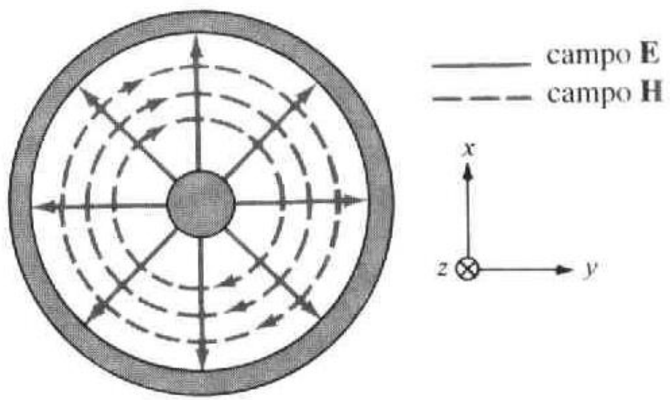
$$\delta = \sqrt{\frac{2}{\mu\omega\sigma}}$$

Linhas de transmissão

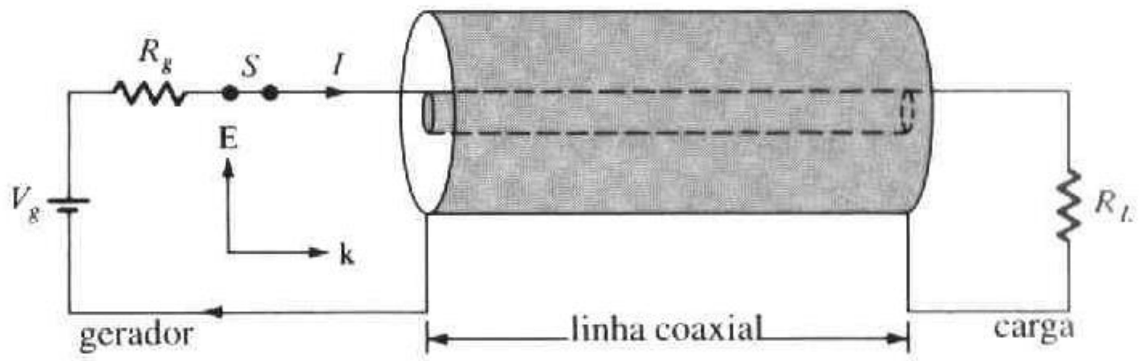


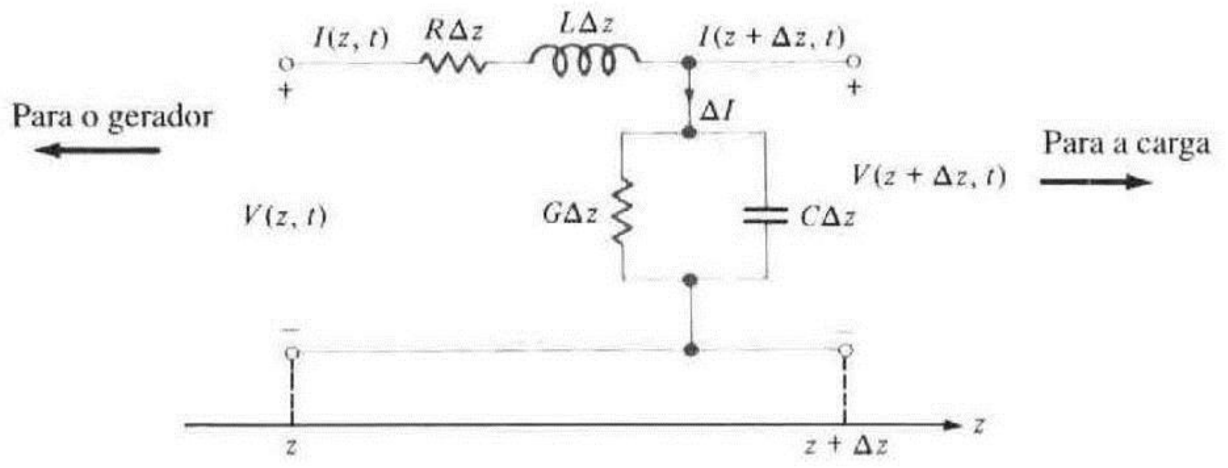
Linhas de transmissão



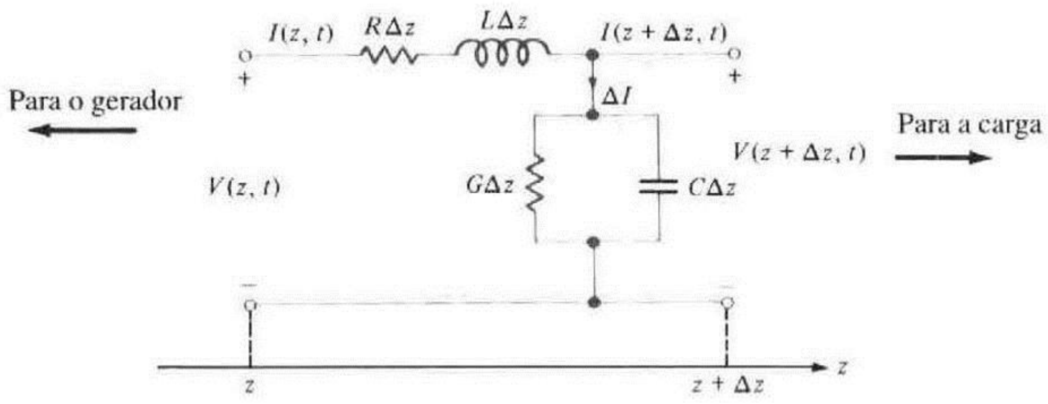


Linhas de transmissão





Linhas de transmissão

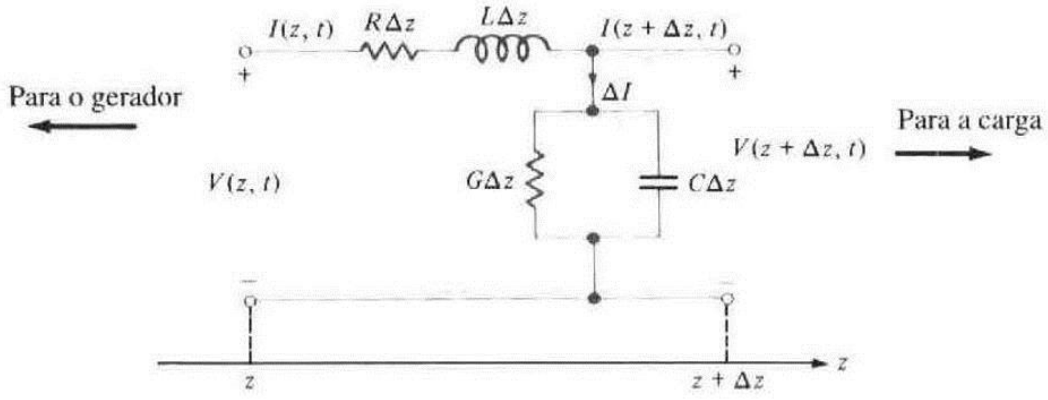


$$V(z,t) = \Delta z R I(z,t) + \Delta z L \frac{\partial I(z,t)}{\partial t} + V(z+\Delta z,t)$$

$$-\frac{V(z+\Delta z,t) - V(z,t)}{\Delta z} = RI(z,t) + L\frac{\partial I(z,t)}{\partial t}$$

$\Delta z \to 0$ $$-\frac{\partial V(z,t)}{\partial z} = RI(z,t) + L\frac{\partial I(z,t)}{\partial t}$$

Linhas de transmissão



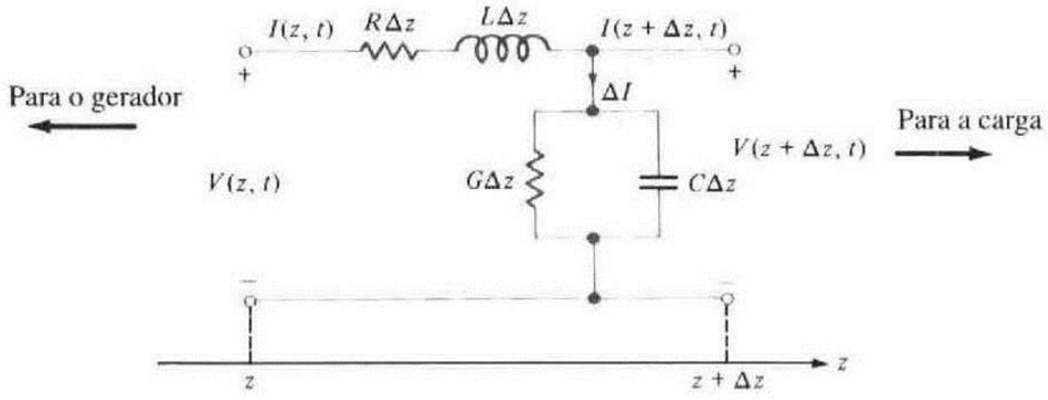
$$I(z,t) = I(z+\Delta z,t) + \Delta I$$

$$I(z,t) = I(z+\Delta z,t) + \Delta z G V(z+\Delta z,t) + \Delta z C\frac{\partial V(z+\Delta z,t)}{\partial t}$$

$$-\frac{I(z+\Delta z,t) - I(z,t)}{\Delta z} = GV(z+\Delta z,t) + C\frac{\partial V(z+\Delta z,t)}{\partial t}$$

$\Delta z \to 0$ $$-\frac{\partial I(z,t)}{\partial z} = GV(z,t) + C\frac{\partial V(z,t)}{\partial t}$$

Linhas de transmissão



$$-\frac{\partial V(z,t)}{\partial z} = RI(z,t) + L\frac{\partial I(z,t)}{\partial t}$$

$$-\frac{\partial I(z,t)}{\partial z} = GV(z,t) + C\frac{\partial V(z,t)}{\partial t}$$

$$V(z,t) = Re\left[V_S(z)e^{j\omega t}\right]$$

$$I(z,t) = Re\left[I_S(z)e^{j\omega t}\right]$$

$$-\frac{dV_S(z)}{dz} = (R + j\omega L)I_S(z)$$

$$-\frac{dI_S(z)}{dz} = (G + j\omega C)V_S(z)$$

Linhas de transmissão

$$-\frac{dV_S(z)}{dz} = (R + j\omega L)I_S(z)$$

$$-\frac{dI_S(z)}{dz} = (G + j\omega C)V_S(z)$$

$$\frac{d^2V_S(z)}{dz^2} = (R + j\omega L)(G + j\omega C)V_S(z)$$

$$\frac{d^2I_S(z)}{dz^2} = (R + j\omega L)(G + j\omega C)I_S(z)$$

$$\frac{d^2V_S(z)}{dz^2} - \gamma^2 V_S(z)$$

$$\frac{d^2I_S(z)}{dz^2} - \gamma^2 I_S(z)$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$V_S(z) = V_0{}^+ e^{-\gamma z} + V_0{}^- e^{\gamma z}$$

$$I_S(z) = I_0{}^+ e^{-\gamma z} + I_0{}^- e^{\gamma z}$$

Linhas de transmissão

$$V_S(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$I_S(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$V(z,t) = Re\left[V_S(z)e^{j\omega t}\right]$$

$$I(z,t) = Re\left[I_S(z)e^{j\omega t}\right]$$

$$V(z,t) = V_0^+ e^{-\alpha z}\cos(\omega t - \beta z) + V_0^- e^{\alpha z}\cos(\omega t + \beta z)$$

$$I(z,t) = I_0^+ e^{-\alpha z}\cos(\omega t - \beta z) + I_0^- e^{\alpha z}\cos(\omega t + \beta z)$$

$$\lambda = \frac{2\pi}{\beta}$$

$$v = \frac{\omega}{\beta} = f\lambda$$

$$z = \frac{1}{\alpha} \rightarrow \ \textit{Decaimento em} \ \frac{1}{e}$$

Linhas de transmissão

$$V_S(z) = {V_0}^+ e^{-\gamma z} + {V_0}^- e^{\gamma z}$$

$$I_S(z) = {I_0}^+ e^{-\gamma z} + {I_0}^- e^{\gamma z}$$

$$-\frac{dV_S(z)}{dz} = (R + j\omega L) I_S(z)$$

$$-\frac{dI_S(z)}{dz} = (G + j\omega C) V_S(z)$$

$$\gamma {V_0}^+ e^{-\gamma z} + \gamma {V_0}^- e^{\gamma z} = (R + j\omega L)({I_0}^+ e^{-\gamma z} + {I_0}^- e^{\gamma z})$$

$$\gamma {I_0}^+ e^{-\gamma z} + \gamma {I_0}^- e^{\gamma z} = (G + j\omega C)({V_0}^+ e^{-\gamma z} + {V_0}^- e^{\gamma z})$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$Z_0 = \frac{{V_0}^+}{{I_0}^+} = \frac{(R + j\omega L)}{\gamma} = \frac{\gamma}{(G + j\omega C)} = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}} \rightarrow \textit{Impedância Característica}$$

Linhas de transmissão

$$V(z,t) = {V_0}^+ e^{-\alpha z} \cos(\omega t - \beta z) + {V_0}^- e^{\alpha z} \cos(\omega t + \beta z)$$

$$I(z,t) = {I_0}^+ e^{-\alpha z} \cos(\omega t - \beta z) + {I_0}^- e^{\alpha z} \cos(\omega t + \beta z)$$

$$Z_0 = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}}$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$4\beta^4 + (RG - \omega^2 LC)\beta^2 - \omega(GL + RC) = 0$$

$$4\alpha^4 - (RG - \omega^2 LC)\alpha^2 + \omega(GL + RC) = 0$$

$$\lambda = \frac{2\pi}{\beta}$$

$$v(\omega) = \frac{\omega}{\beta(\omega)}$$

$$z(\omega) = \frac{1}{\alpha(\omega)} \rightarrow \textit{Decaimento em } \frac{1}{e}$$

Linhas de transmissão

$$R = G = 0$$

$$Z_0 = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}} = \sqrt{\frac{L}{C}}$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{-\omega^2 LC} = j\omega\sqrt{LC}$$

$$\alpha = 0 \qquad \beta = \omega\sqrt{LC} \qquad v = \frac{1}{\sqrt{LC}}$$

Meio sem perda

Linhas de transmissão

$$RC = GL$$

$$\gamma = \alpha + j\beta \;= \sqrt{\frac{1}{LC}(RC + j\omega LC)(GL + j\omega LC)} \;= \sqrt{\frac{1}{LC}}(RC + j\omega LC) = \sqrt{RG} + j\omega\sqrt{LC}$$

$$\alpha = \sqrt{RG} \qquad\qquad \beta = \omega\sqrt{LC}$$

$$Z_0 = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}} = \sqrt{\frac{R}{G}} = R + jX \qquad\qquad X = 0$$

Meio sem dispersão